Principal component analysis of the nine dimensions of national power of seventy-four countries

Norma Raquel Guzman-Mendieta[1], **Jesús Antonio Guerrero-López**[1], **Marcelo del Castillo-Mussot**[2,*], **Eric Hernández-Ramírez**[3], **Reyhane Salehabadi**[4],
1 Facultad de Ciencias, Universidad Nacional Autónoma de México, Mexico City, Mexico
2 Instituto de Física, Universidad Nacional Autónoma de México, Mexico City, Mexico
3 Instituto de Investigaciones Económicas, Universidad Nacional Autónoma de México, Mexico City, Mexico
4 Tarbiat Modares University, Iran

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* mussot@fisica.unam.mx

Abstract

National power is not a single quantity but a complex web of economic, political, and territorial capacities that evolve together. In this study, we examine 74 countries through the "power pyramids" framework developed by Salehabadi *et al.*, which integrates nine dimensions of power— territorial, economic, political, scientific–technological, cultural, social, military, transboundary, and spatial—measured with 314 standardized indicators.

Following the original scaling procedures, we conduct a cross–country correlation analysis that reveals strong associations between economic and technological strength, as well as between social development and political stability. These relationships point to deep structural dependencies among different aspects of national capability. To explore these dependencies further, we apply Principal Component Analysis (PCA), which reduces the ninefold structure into a few interpretable axes while preserving most of the original variance.

Two principal dimensions emerge as dominant. The first represents a general capacity gradient in which economic, technological, and cultural resources underpin global influence. The second contrasts internal governance with external projection, distinguishing states that emphasize cohesion and institutional development from those that rely on territorial and military power.

Together, these findings suggest that national power should be understood as an interconnected system that nonetheless can be represented in a simpler two–dimensional framework. This approach provides a clear comparative lens to understand how countries shape distinct profiles and strategies in global politics.

Introduction

The study of national power has long been a central concern in geopolitics and international relations. Power shapes the ability of states to pursue their interests, secure their territory, and influence outcomes in the international system. Yet despite its importance, defining and measuring power remains a deeply contested task. In its

broadest sense, national power refers to a state's capacity to shape international processes, achieve its objectives, and protect them against internal or external pressures. This capacity draws on a combination of material, institutional, and symbolic resources that vary across contexts. In practice, these resources constitute the observable manifestations of power and provide the basis for systematic evaluation [1].

Scholars have long debated the proper foundations of such evaluation. Early approaches often adopted a narrow economic perspective, relying on measures such as gross domestic product (GDP) or gross national product (GNP) as the principal indicators of comparative capability [2–4]. While appealing in their simplicity, these unidimensional metrics have been widely criticized for overlooking other sources of influence that shape state behavior.

In response, numerous authors have advocated for multidimensional perspectives. Waltz [5] emphasized population size, territory, natural resources, military capacity, and institutional quality as key determinants of a state's standing in the international system [6]. Later contributions by Mearsheimer [7] and Sułek [8] highlighted the enduring relevance of military strength, even as technological innovation and nuclear capabilities transformed strategic realities.

These debates reveal a persistent tension between parsimony and complexity in the measurement of national power. Simple indicators allow for comparability but risk oversimplifying; richer frameworks capture multiple dimensions of power but raise difficult questions about indicator selection, weighting, aggregation, and replicability. In recent decades, scholars have sought to reconcile these tensions by proposing multidimensional models that integrate economic, military, political, demographic, cultural, and technological components. While such models offer a broader view, they often differ significantly in scope, transparency, and methodological consistency.

One of the most ambitious contributions to this literature is the framework developed by Salehabadi et al. [9]. Drawing on more than eighty theoretical models, the authors identify common features and methodological limitations before presenting a comprehensive structure composed of nine dimensions: territorial, economic, political, military, scientific–technological, cultural, social, transboundary, and spatial. Importantly, the model embraces both traditional sources of power and emerging domains such as outer-space capability.

A notable strength of their contribution is the combination of conceptual breadth with systematic implementation. The authors developed software capable of organizing, analyzing, and visualizing large volumes of data, producing tables, graphs, maps, and hierarchical "power pyramids" that facilitate cross-national comparison. From over 400 initial indicators compiled from international databases, institutional reports, expert assessments, and prior research, the authors retained 314 variables that satisfied criteria of coverage, quantifiability, relevance, and reliability.

Within each dimension, factor analysis was applied to isolate the most influential indicators, retaining those that collectively explained at least 50% of the variance. Four scoring strategies were used to manage heterogeneity among variables: proportional shares, Morris index rescaling, benchmarking against reference values, and the integration of standardized indices such as HDI or GII.

An important methodological feature of their framework is the weighting and normalization process. As explained by the authors, the initial weights were derived from expert judgments from specialists inside and outside Iran. These expert-based weights were combined with factor-loading results to identify the most important contributors in each dimension, while indicators with low importance coefficients were removed. The normalization strategy explicitly distinguishes between variables with positive and negative effects on national power: measures such as cultural cohesion enter the model with positive loadings, whereas variables with detrimental

implications—such as inflation—enter with negative loadings. The final aggregated values for each dimension are computed using an ideal-programming scheme, which integrates these signed contributions into a coherent evaluative structure.

Taken together, this methodological architecture gives the Salehabadi et al. model a rare combination of theoretical rigor, empirical detail, and computational formalization. It stands as one of the most extensive and systematically constructed multidimensional measures of national power currently available, and provides a solid foundation for further analytical development.

The Makarov model

The Salehabadi *et al.* [9] model is not the only recent attempt to capture the multidimensionality of power. Makarov and colleagues [10–12] developed an "integral index of national power" that aggregates standardized indicators across socioeconomic, demographic, scientific, and geopolitical domains. Their model, though different in scope, shares the aim of conceptualizing power as a complex system of interdependent resources and capacities. Together, these frameworks establish a robust foundation for empirical comparison.

Building on this foundation, the present study does not seek to replicate the rankings generated by Salehabadi *et al.* [9]. Instead, it uses their dataset to explore the internal structure of national power. Specifically, we focus on the eight substantive components—territorial, economic, political, military, scientific–technological, cultural, social, and transboundary—treating the aggregate "global power" index as a synthetic outcome rather than an independent dimension.

Our contribution is twofold. First, we conduct correlation analysis to examine the interdependencies among the components, identifying overlaps and synergies that may be obscured in the original framework. Second, we apply Principal Component Analysis (PCA) to extract latent patterns that organize countries within this multidimensional space. By reducing complexity while preserving explanatory power, PCA enables us to visualize how countries align along principal axes of national capability and influence.

In this way, we bring a complex-systems perspective to the study of power. Power is not merely the sum of separate resources; it emerges from interactions and combinations that amplify or constrain a country's overall capacity. PCA provides a statistical means to uncover these hidden structures, offering a more nuanced interpretation of how different forms of power co-evolve and shape global hierarchies.

Table 1. Countries with the highest number of minimum scores (1) across power dimensions.

Country	Total number of 1s
United States	5
Russia	1
Switzerland	1
Denmark	1
Sweden	1
Finland	1
Thailand	0
Portugal	0
Bulgaria	0
Croatia	0

Materials and methods

Our analysis combines two complementary statistical techniques: correlation analysis and Principal Component Analysis (PCA). Correlation analysis quantifies the strength and direction of linear relationships between variables, while PCA reduces the dimensionality of the dataset by transforming correlated variables into a smaller set of orthogonal components that retain most of the original variance. Both methods are well established in multivariate statistics and widely used in socio-economic and geopolitical research.

Correlation

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, and let $X, Y \in L^2(\Omega, \mathcal{F}, \mathbb{P})$ be two real-valued random variables with finite second moments. The *Pearson correlation coefficient* [13, 14] is defined as:

$$\rho_{XY} = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\mathbb{E}[(X-\mu_X)(Y-\mu_Y)]}{\sqrt{\mathbb{E}\left[(X-\mu_X)^2\right]\ \mathbb{E}\left[(Y-\mu_Y)^2\right]}}, \tag{1}$$

where $\mu_X = \mathbb{E}[X]$ and $\mu_Y = \mathbb{E}[Y]$ are the expected values, and $\sigma_X, \sigma_Y > 0$ are the standard deviations of X and Y.

The coefficient ρ_{XY} satisfies:

$$-1 \leq \rho_{XY} \leq 1, \tag{2}$$

where $\rho_{XY} = 1$ indicates perfect positive dependence, $\rho_{XY} = -1$ indicates perfect negative dependence, and $\rho_{XY} = 0$ denotes orthogonality in L^2, implying that X and Y are uncorrelated but not necessarily statistically independent unless jointly Gaussian.

Principal Component Analysis (PCA)

Let $\mathbf{X} \in \mathbb{R}^{n \times p}$ be a data matrix with n observations (rows) and p variables (columns). We assume that $\mathbf{X}$ is column-centered, i.e.,

$$\frac{1}{n}\sum_{i=1}^{n} x_{ij} = 0 \quad \text{for all } j = 1, \ldots, p. \tag{3}$$

The *sample covariance matrix* is defined as:

$$\mathbf{S} = \frac{1}{n-1}\mathbf{X}^\top \mathbf{X}. \tag{4}$$

PCA [15, 16] is based on the spectral decomposition:

$$\mathbf{S}\mathbf{u}_k = \lambda_k \mathbf{u}_k, \quad k = 1, \ldots, p, \tag{5}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$ are the eigenvalues, and $\mathbf{u}_k \in \mathbb{R}^p$ are the orthonormal eigenvectors (*principal directions*).

The k-th *principal component score vector* is given by:

$$\mathbf{z}_k = \mathbf{X}\mathbf{u}_k, \tag{6}$$

with variance $\mathrm{Var}(\mathbf{z}_k) = \lambda_k$. The proportion of total variance explained by each component is:

$$\mathrm{EVR}_k = \frac{\lambda_k}{\sum_{j=1}^{p} \lambda_j}, \tag{7}$$

which measures the contribution of the k-th component to the overall variability in the dataset.

Matrix construction

The empirical dataset was derived from the "power pyramids" of national power presented by Salehabadi *et al.* [9]. Each pyramid assigns ordinal values from 1 (highest) to 5 (lowest) for each country across nine fundamental dimensions of power: territorial, economic, scientific–technological, social, cultural, political, military, transboundary, and spatial. The complete classification can be consulted in the original publication.

We extracted the values provided by the authors and organized them into a structured matrix, where each row corresponds to a country and each column to a power dimension. This transformation converted the qualitative assessments into a consistent numerical form, making each dimension a quantitative variable suitable for correlation analysis.

Once the matrix was assembled, we computed the Pearson correlation coefficients for all pairs of dimensions. The resulting correlation matrix captures the degree of linear association among components of national power—for example, how economic strength relates to scientific–technological development or to political and military capability. The ordinal scores (1–5) used for each country were directly obtained from the power pyramids published by Salehabadi *et al.* [9], where the full classification of national power dimensions is available.

To illustrate the data structure used to construct the correlation matrix, Table 2 presents a short excerpt of the dataset derived from the "power pyramids" classification of Salehabadi *et al.* [9]. Each row represents a country and each column one of the nine dimensions of national power. The entries are ordinal values ranging from 1 (higher capacity) to 5 (lower capacity), indicating each country's relative position in that dimension. For brevity, only five representative countries are shown here, while the complete classification covering 74 countries can be consulted in Salehabadi *et al.* [9].

Table 2. Illustrative excerpt (first five countries).

Country	Terr.	Econ.	Sci-Tech	Soc.	Cult.	Pol.	Mil.	Transb.	Space
Russia	1	4	4	5	4	5	2	2	2
China	2	2	3	5	4	5	2	2	2
United States	2	1	1	3	2	3	1	1	1
Canada	2	3	3	3	2	2	4	2	3
Brazil	2	5	4	5	4	5	3	4	4

Notes: Ordinal classifications (1–5) across nine power dimensions (Terr., Econ., Sci-Tech, Soc., Cult., Pol., Mil., Transb., Space). Values are taken from the "power pyramids" reported by Salehabadi *et al.* [9]. Only five countries are shown here for illustration; the original study covers 74 countries in total.

The correlation structure reveals the interdependencies among the nine power dimensions and provides the basis for the subsequent PCA. Applying PCA to this matrix reduces the dataset to a smaller number of components that capture the dominant patterns of variation. This step helps uncover latent structures and clusters among countries, offering insight into how different forms of power interact within the global system. For clarity, throughout the tables and figures we use the following abbreviations: Terr. (territorial), Econ. (economic), Sci-Tech (scientific–technological), Soc. (social), Cult. (cultural), Pol. (political), Mil. (military), Transb. (transboundary), and Space (spatial).

Results and discussion

The correlation matrix provides an initial view of how the nine dimensions of national power move together. Territorial power is positively associated with military ($\rho = 0.57$),

transboundary ($\rho = 0.56$), and spatial power ($\rho = 0.63$), suggesting that countries with extensive territorial capacity also tend to project influence through armed forces, cross-border ties, and control in maritime or aerospace domains. Economic power emerges as a central hinge, correlating strongly with scientific–technological capacity ($\rho = 0.83$), cultural power ($\rho = 0.72$), and transboundary activity ($\rho = 0.65$). This pattern supports the idea that economic development underwrites knowledge production, facilitates cultural diffusion, and sustains a broader global footprint. Scientific–technological capacity shows the highest connectivity overall (e.g., $\rho = 0.81$ with cultural power), reinforcing its cross-cutting role. Social and cultural power are closely linked ($\rho = 0.76$) and both relate tightly to political power ($\rho = 0.79$ and $\rho = 0.78$), highlighting how domestic cohesion, cultural production, and institutional performance often co-evolve. Finally, military and spatial power are strongly connected ($\rho = 0.70$), and transboundary power is even more closely associated with spatial power ($\rho = 0.72$), pointing to complementarities between global reach, geographical positioning, and security projection.

To summarize the overall dimensional structure of the data, we applied Principal Component Analysis (PCA) to the correlation matrix derived from the classification of Salehabadi et al. [9]. This procedure identifies the main axes that organize the relationships among the nine power dimensions, allowing us to represent most of the information in a reduced number of components. The first two principal components account for nearly 80% of the total variance, indicating that much of the complexity in national power can be represented in two dimensions. Table 3 summarizes the proportion of variance explained by each component, while Table 4 details the variable loadings that define their interpretation.

Table 3. Explained variance of the principal components. The first two components together capture almost 80% of the total variance, providing a compact summary of the multidimensional structure of national power.

Component	Explained variance (%)	Cumulative variance (%)
PC1	56.87	56.87
PC2	22.22	79.09
PC3	5.60	84.69
PC4	4.14	88.83
PC5	3.76	92.58
PC6	2.48	95.07
PC7	2.09	97.16
PC8	1.66	98.82
PC9	1.18	100.00

Table 4. Loadings of the variables on the principal components. Values represent correlations between each original variable and the corresponding component. Loadings above $|0.50|$ are considered salient for interpretation.

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
Territorial power	-0.236	-0.445	0.691	-0.336	-0.239	-0.101	0.222	0.190	0.056
Economic power	-0.379	0.088	-0.232	0.367	-0.650	0.104	0.221	0.087	0.412
Scientific-technological	-0.406	0.089	-0.215	-0.133	-0.274	-0.290	0.067	-0.100	-0.768
Social power	-0.316	0.392	-0.078	-0.232	0.466	0.006	0.641	0.200	0.136
Cultural power	-0.383	0.229	0.002	-0.140	0.098	-0.200	-0.650	0.531	0.168
Political power	-0.330	0.373	0.306	-0.214	-0.025	0.293	-0.235	-0.672	0.135
Military power	-0.218	-0.502	-0.492	-0.441	0.068	0.499	-0.085	0.000	0.050
Transboundary power	-0.364	-0.175	0.242	0.616	0.331	0.427	-0.038	0.123	-0.300
Spatial power	-0.317	-0.398	-0.147	0.202	0.322	-0.581	-0.022	-0.397	0.282

The first principal component (PC_1) loads negatively across all nine dimensions, with the strongest contributions from economic (−0.38), scientific–technological (−0.41), cultural (−0.38), political (−0.33), and transboundary (−0.36) power. Social (−0.32) and spatial (−0.32) dimensions also load substantially, while military (−0.22) contributes slightly less. Taken together, PC_1 reflects a broad axis of general capacity: countries with higher scores on this dimension tend to combine multiple forms of power—economic, knowledge-based, political, cultural, and territorial. For interpretability, the sign of this component was inverted so that higher values correspond to greater overall capacity.

The second component (PC_2) captures a sharper contrast. Positive loadings are associated with political (0.37), social (0.39), and cultural (0.23) power, while negative loadings correspond to territorial (−0.44), military (−0.50), and spatial (−0.40) dimensions. This axis distinguishes countries that emphasize social and political cohesion from those that project influence through territorial control, military reach, and spatial expansion.

Fig. 1 displays the distribution of countries in the PC_1–PC_2 plane. Each point represents a country's score in the principal-component space, and arrows (if shown) indicate the orientation of variable loadings. The horizontal axis corresponds to the general capacity gradient, while the vertical one highlights the balance between internal cohesion and external projection.

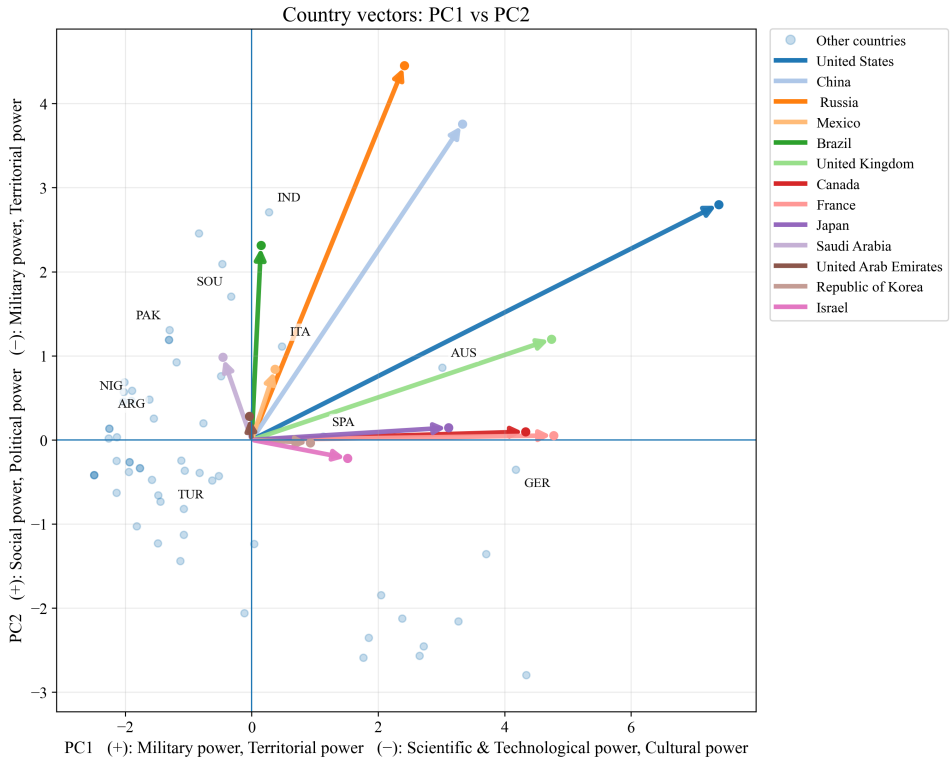


Fig 1. Biplot of countries on the first two principal components (PC_1 vs. PC_2). Each point represents a country. The horizontal axis (PC_1) shows a general capacity gradient, while the vertical axis (PC_2) contrasts socio–political cohesion with territorial–military projection. Arrows, when present, depict the direction and magnitude of variable loadings.

As illustrated in Fig. 1, the United States, China, and Russia extend far along the positive side of PC_1, reflecting broad capabilities across economic, scientific, and

geopolitical dimensions. In contrast, countries such as Mexico or Saudi Arabia project more along PC_2, emphasizing domestic cohesion and political consolidation rather than external expansion. Mid-range states such as India and Brazil appear near the center, balancing internal governance with moderate global projection, while smaller or regionally constrained powers cluster toward the lower-left quadrant. Together, these two components explain 79.1% of the total variance, summarizing much of the structural diversity in the dataset and revealing how the interplay between capacity and cohesion defines the first layer of global hierarchy.

Beyond this two-dimensional map, additional components reveal finer distinctions. Fig. 2 presents the biplot of PC_1 and PC_3. Here, PC_3 differentiates alternative bases of hard power—positive values emphasize military and economic strength, whereas negative values highlight territorial extension and institutional stability. The PC_1–PC_3 plane further separates states that combine strong economic and military capabilities—visible in the upper-right quadrant—from those whose influence derives from territorial size or governance structures, such as India or Brazil, which appear toward the lower range. This contrast suggests that material power can rest either on concentrated hard-power synergy or on extensive territorial–institutional capacity.

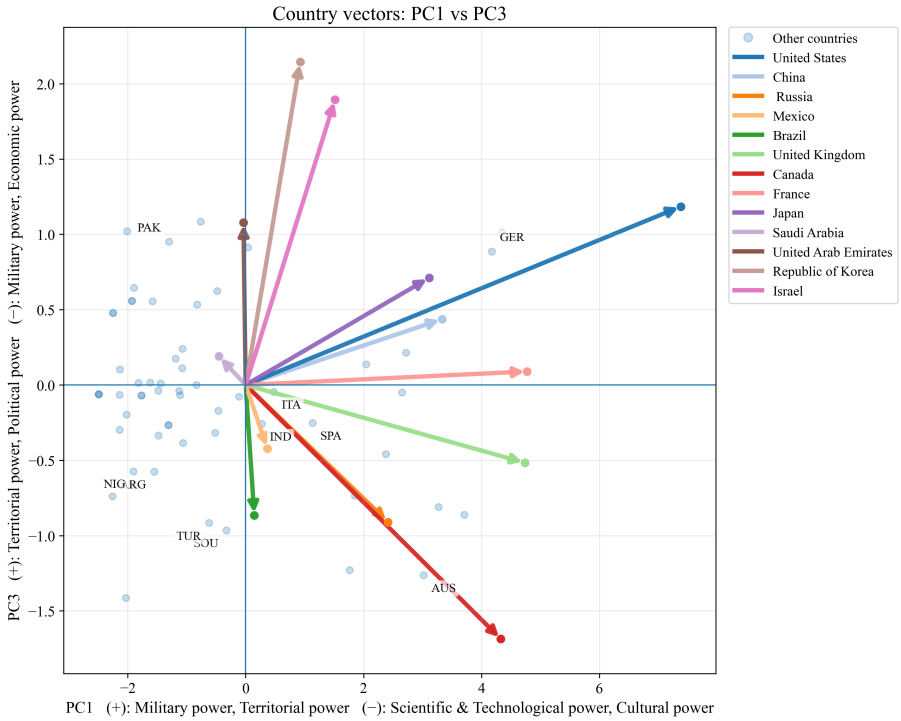


Fig 2. Biplot of countries on PC_1 and PC_3. Each point corresponds to a country. PC_1 captures overall capacity, while PC_3 separates hard-power bases: military–economic emphasis (positive) versus territorial–institutional foundations (negative). Arrows, when shown, indicate variable loadings.

Finally, Fig. 3 explores a subtler contrast between material and symbolic dimensions of power. While PC_1 again represents overall capacity, PC_8 distinguishes between countries deriving influence from cultural legitimacy and symbolic recognition (positive) and those emphasizing spatial and political assertion (negative). The PC_1–PC_8 projection isolates this ideational dimension: countries such as Japan and Israel occupy the positive end, reflecting strong cultural and technological soft power, whereas those prioritizing geopolitical control and spatial dominance (e.g., Russia, China) cluster

toward the negative side. This component therefore bridges material and symbolic power, offering a perspective on how states convert economic and technological assets into international prestige or strategic influence.

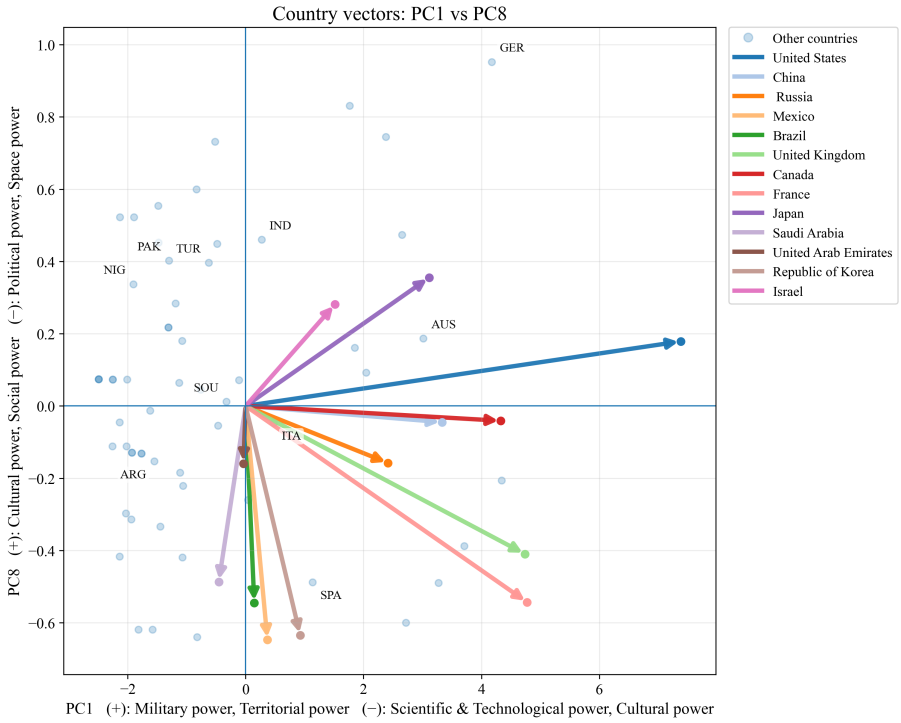


Fig 3. Biplot of countries on PC_1 and PC_8. Each point represents a country. The horizontal axis (PC_1) indicates general capability, while the vertical axis (PC_8) distinguishes cultural prestige and symbolic influence (positive) from spatial and political assertion (negative). Arrows, when present, denote variable loadings.

Taken together, the three biplots reveal a layered structure of national power. PC_1 summarizes overall capacity, PC_2 reflects a governance–versus–projection contrast, PC_3 refines distinctions among hard-power foundations, and PC_8 captures symbolic and cultural influence. This multidimensional pattern illustrates how states balance material resources, institutional cohesion, and soft-power appeal to define their distinct profiles within the international system.

Conclusions

The results point to two central insights. National power emerges as an interdependent system in which economic, technological, and cultural resources provide the foundations of global influence, while territorial and military capacities act as means of external projection. At the same time, this complexity can be summarized through two main dimensions: a broad *capacity gradient* and a *governance–versus–projection* axis. Together, they offer a concise way to compare countries and to understand the distinctive profiles they trace within the international order.

Beyond these two dominant components, higher-order patterns add depth to the interpretation. PC_3 differentiates between countries that rely on concentrated military–economic strength and those whose influence rests on territorial reach and institutional stability. PC_8, in turn, contrasts symbolic and cultural forms of influence

with spatial and political assertion, capturing how prestige, legitimacy, and recognition complement material power. These additional axes underscore that national power is not monolithic but layered, mixing material, institutional, and symbolic elements.

Methodologically, the study also contributes by demonstrating how correlation analysis and PCA together reveal the latent architecture of power. Correlation coefficients expose the direct links among dimensions, whereas PCA condenses those relationships into a smaller number of interpretable axes that reflect both general capacity and strategic orientation. This combination provides a transparent and reproducible framework for cross-country comparison.

Finally, the analysis opens several paths for future work. Tracking how countries move within the PCA space over time could reveal trajectories of rise, decline, or transformation in their power configurations. Extending the framework to include new variables—such as environmental sustainability, digital infrastructure, or soft-power indicators—would enrich the picture. More broadly, this multidimensional and data-oriented approach strengthens the dialogue between international relations theory and quantitative modeling, offering a coherent way to study how states build, project, and sustain power in a changing global landscape.

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